TECH NOTE:

CALCULATING MAX ACCELERATION FOR EXLAR® ACTUATORS

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Terminology

α = Angular acceleration ($\mathrm{rad/s^2}$)

μ = System friction coefficient (unitless)

η = Efficiency (unitless)

F = Linear force (N)

i = Maximum current (A)

J = Inertia ($\mathrm{kgm^2}$)

K_t = Torque constant ($\frac{\mathrm{Nm}}{\mathrm{A}}$)

L = Screw lead (m)

m = Mass (Kg)

N = Drive ratio (unitless)

S = Stroke increments (mm)

T = Torque (Nm)

ta = Time for acceleration

tm = Time of max acceleration

1. Max Acceleration

Acceleration is the rate of change of velocity with respect to time. To evaluate maximum acceleration for a linear actuator, it can help to visualize a 0-60 mph time for a car. To determine the maximum acceleration for an application, a detailed motion profile is required to understand the following:

1. *Distance of the move*
2. *Time to complete the move*
3. *Mass of the load*
4. *Total stroke length available*
5. *Friction loss*
6. *Operational temperature*
7. *Duty cycle*

Manufacturers are interested in reducing costs and improving throughput. Improving throughput requires that actuators move as fast as possible. However, the maximum velocity of a linear actuator is not the only contributing factor to the overall rate at which a move can be completed. The maximum rate of acceleration of the system (actuator + tooling + load) should also be considered, i.e., the quicker the acceleration, the less total time required to complete desired move(s).

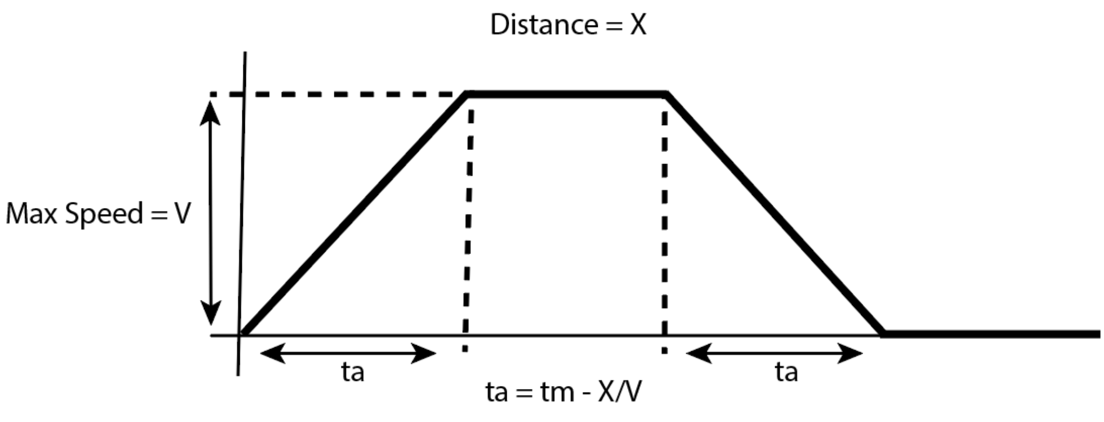


Figure 1: Trapezoidal Move Profile (mm/s)

2. Formulas

Maximum acceleration is dependent on the **Inertia** of all moving components. Inertia is a conversion factor that determines how much torque is required to accelerate an object based on its mass and shape. Inertias are additive; therefore, the total inertia of a system can be broken down into its various components:

$$J_{total} = J_{Linear} + J_{load} + J_{Rotary} + J_{Actuator} + J_{etc.}$$

Not all these components are present in every system, and each has a unique formula. For rotary units (e.g. R2M/G, SLM/G):

$$J_{Rotary} = J_{Base} + J_{gearbox}$$

For linear actuators (e.g GTX, FTX):

$$J_{Linear} = \frac{(J_{Base} + S * J_{Stroke})}{N^2}$$

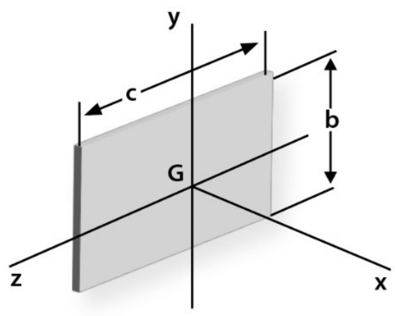
The J_{Base}, J_{stroke}, and $J_{gearbox}$ components are available for each Exlar product on the website. N will be 1 for all lines except KX and FTX. For the latter, N=2 if using a 2:1 pulley.

Inertia of a **linear** load (including tooling mass)

$$J_{Load} = m_{Load} * \left(\frac{L}{2\pi N}\right)^2$$

Figure 2 lists a variety of inertia equations for **rotational** moments of the load. Note: these assume no reduction, i.e., N=1. If there is a reduction, divide by N^2.

Thin Rectangular Plate:

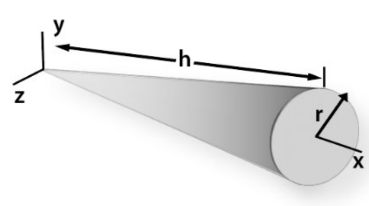


$$Jx = \frac{1}{12}m(b^2 + c^2)$$

$$Jy = \frac{1}{12}mc^2$$

$$Jz = \frac{1}{12}mb^2$$

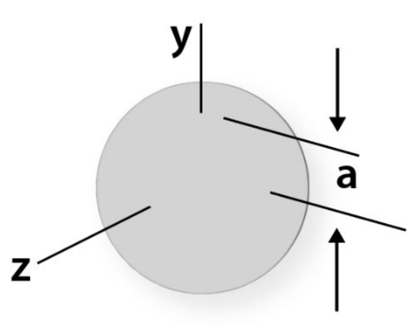
Circular Cone:



$$Jx = \frac{3}{10}ma^2$$

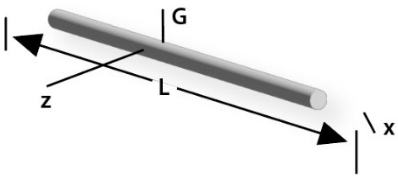
$$Jy = Jz = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$$

Sphere:



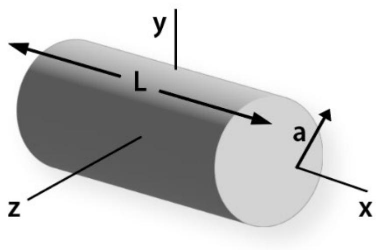
$$Jx = Jy = Jz = \frac{2}{5}ma^2$$

Slender Rod:



$$Jy = Jz = \frac{1}{12} mL^2$$

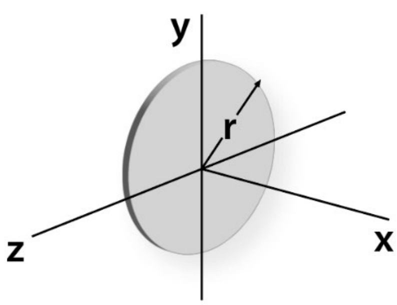
Circular Cylinder:



$$Jx = \frac{1}{2} ma^2$$

$$Jy = Jz = \frac{1}{12} m(3a^2 + L^2)$$

Thin Disk:



$$Jx = \frac{1}{2} mr^2$$

$$Jy = Jz = \frac{1}{4} mr^2$$

Figure 2: Inertia Equations

3. Torque

Torque is required to drive motion of the system and complete the desired work, i.e., moving applied load(s). Torque of a servo system is proportional to the input current to the system. The more toque that can be applied to completing the desired work, the smaller and lower cost the system can be designed.

Therefore, system designers should optimize the acceleration torque to meet the desired motion profile without oversizing the motion system. A common way to reduce acceleration torque is to reduce the load inertia. If the inertia ratio is too high, decrease the load inertia by:

- Decreasing the lead of the screw (L is a squared term for inertia)
- Use a smaller actuator or motor (r is a squared term for inertia)
- Decrease the load mass
- Add a gearbox (N is squared term for inertia)

Total Torque = Maximum Theoretical Torque output of the motor

$$T_{total} = T_{actuator} + T_{Friction} = K_t * i$$

Friction Torque = Constant proportional to coefficient of friction of the system plus the actuator's internal friction (published in catalog).

$$T_{Friction} = \mu * T_{actuator} + T_{actuator\ friction}$$

Acceleration Torque = Constant, only present during acceleration and deceleration.

$$T_{actuator} = J_{Total} * \frac{\alpha}{\eta}$$

Thus **maximum acceleration** for a **Linear** load is:

$$\alpha = \frac{\eta * (K_t * i - T_{f\ actuator})}{\left(\frac{(J_{actuator} + S * J_{Stroke})}{N^2} + (\frac{2\pi}{L})^2 * m + (J_{motor} + J_{gearbox})\right) * (1 + \mu)}$$

All values except load mass and system friction can be found in the Exlar product catalog. Load mass and system fricition need to be considered in each motion system design as they are specific to each unique application.

For Integrated/Intelligent (e.g. GTX, TTX) actuators, this simplifies to

$$\alpha = \frac{\eta * (K_t * i + T_{f\ actuator})}{\left(J_{base} + (S * J_{Stroke}) + (\frac{2\pi}{L})^2 * m\right) * (1 + \mu)}$$

This gives the answer in $radians/sec^2$

- To convert to deg/sec^2, multiply by $180/\pi$
- To convert to rev/sec^2, divide by $2 * \pi$
- To convert to in/sec^2 or m/sec^2, mulitply by $L/(2 * \pi)$ where L is the screw lead in either inches or meters, respectively.

For optimized motion & overall system performance, it is best to ensure the actuator and load have similar inertia values, also referred to as inertia matching. (J_{Load}: $J_{Actuator/motor}$) Some general guidelines below and in figure 3:

- *10:1 Maximum acceptable inertia ratio*
- *5:1 typical*
- *2:1 or less for highest performance (rigidity/resonance)*
- *1:1 will grant the maximum acceleration*

Gear Ratio	Torque Out	Speed Out	Load Inertia at Motor
1:1	1*e	1	1
2:1	2x*e	1/2	1/4
5:1	5x*e	1/5	1/25
n:1	nx*e	1/n	$1/n^2$

Figure 3: Optimal Configurations

It may be tempting to aim for less than 1:1 by increasing the motor inertia, but keep in mind angular acceleration is dependent on total system inertia, so instead of the load inertia being the limiting factor, it will now be the motor.

The gear ratio that will grant maximum acceleration is as follows (note: gearboxes are only available on our rotary products. This formula applies to combinations of universal and rotary actuators).

$$N = \sqrt{\frac{m_{Load} * (\frac{L}{2\pi})^2 + (J_{BaseUniversal} + S * J_{Stroke})}{J_{BaseRotary} + J_{gearbox}}}$$

We offer gearboxes with a variety of reductions ranging from 4:1 to 100:1 on most motors as well as 2:1 belt reduction on all actuators. Choose the ratio that is the closest to the value for N. Sizing up a gearbox may not always result in higher acceleration. Based upon gear set geometry it is possible to overload the gearing causing damage. Ensure when selecting an appropriate gearset to not exceed maximum rated torque values.

Please keep in mind that optimizing for maximum acceleration may impact performance in other areas, such as maximum force or velocity. To optimize your next linear motion application please contact Exlar's Application Engineering department at CHA_Applications@curtisswright.com

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